AVERAGE PATH LENGTH IN SMALL-WORLD NETWORKS

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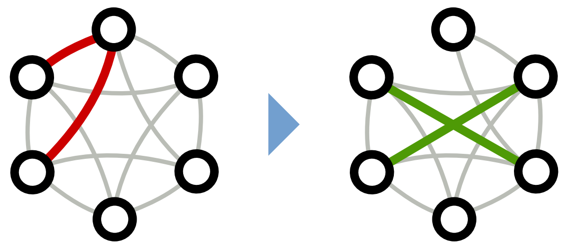
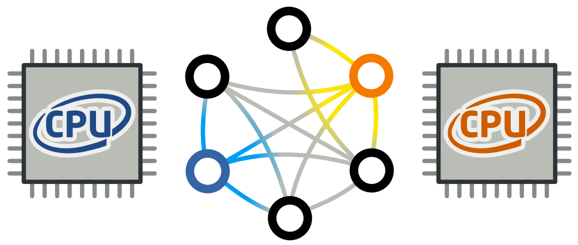
The Algorithm:

1



Create a Small-World network by randomly reconnecting some edges of a ring lattice network (the Watts-Strogatz model)

2

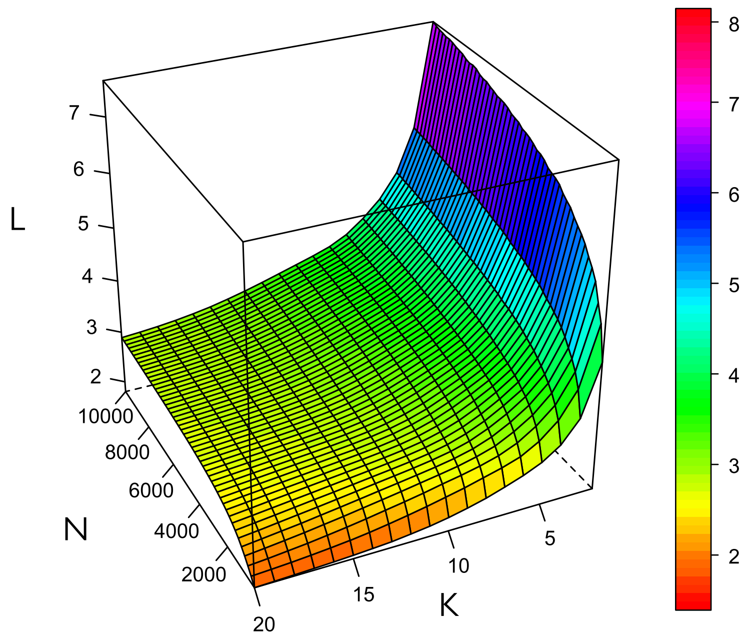


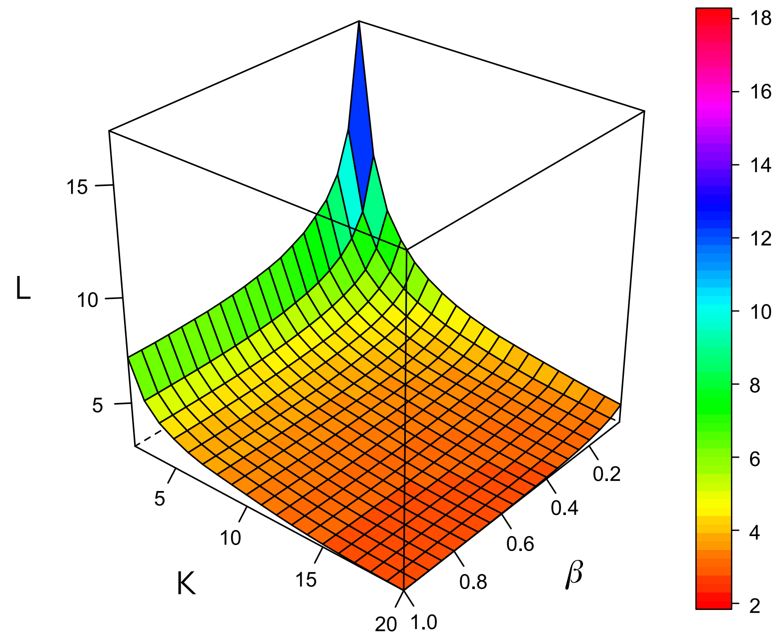
Using *Open-MP*, perform a breadth-first-search from every node using multiple CPUs. Sum the minimum path-lengths to every node.

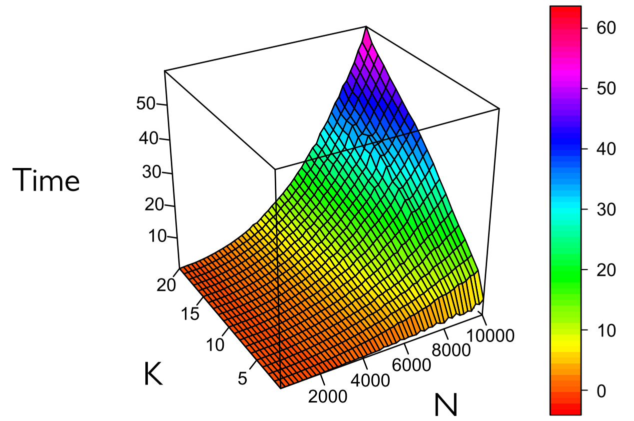
3

$$L = \frac{\sum_{i,j} d(v_i, v_j)}{n(n-1)}$$

Divide the summed shortest-path lengths by the number of paths to find the average path length.







Experimental Results

Top Left: Average path length for β = 0.5

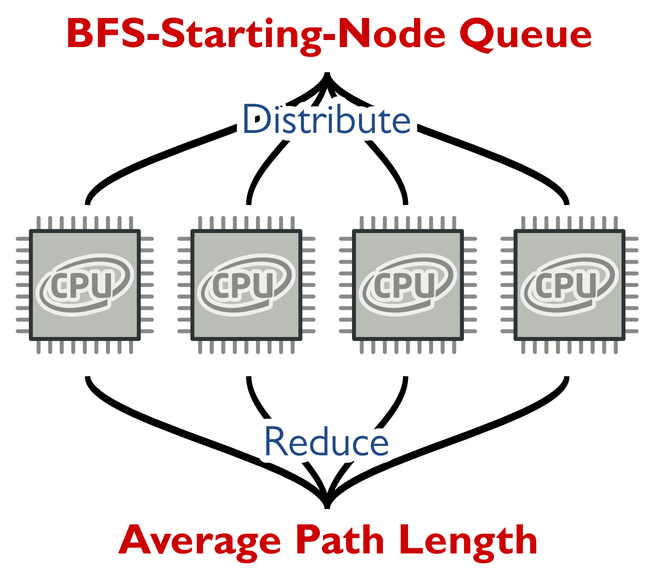
Top Right: Average path length for N = 10000

Bottom Left: Performance landscape for β = 0.5. This confirms the given time complexity

L: Average Path Length; N: Number of Nodes; K: Number of Neighbours; β: Reconnect Probability

Parallelisation

One way to facilitate a BFS search on a large graph is to execute the search in parallel. We begin by creating a team of worker threads one per logical CPU. Work is then sent to the threads to be executed. In our case, each thread executes a BFS origininating from a different node. All threads share the same graph structure in memory but have an individual search state.



Vs. Floyd's Algorithm

Performance determined by the number of vertices in the graph running in $O(V^3)$. This experiment uses a small-world network - this has, by definition, a large number of vertices with a relatively low number of edges between them and so this is not helpful for this algorithm.

Vs. Dijkstra's Algorithm

Dijkstra's algorithm outperforms Floyd's due to the significant term being $O(V \log, V)$ not $O(V^3)$. An every-source dense graph problem is solved in $O(\frac{1}{2}V^2(V + 2\log V - 1))$ - marginally faster than Floyd's. In practice, BFS suits the given problem better, as edge weights are constant and complexity being $O(EV)$. Since $E = KN$, $V = N$, we have $O(KN^2)$.

References

Duncan J. Watts & Steven H. Strogatz; "**Collective dynamics of 'small-world' networks**" Nature Volume 393, 4 June 1998. pp 440-442.

OpenMP Architecture Review Board; "**OpenMP Application Program Interface**" Version 3.1, July2011